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Quantum linear magnetoresistance; solution of an old mystery

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Abstract

In this paper, the history of the discovery of the linear magnetoresistance in metals by Kapitza from 1928–1929 and its explanation are described. Actually, Kapitza discovered two different phenomena. One of them, the linear magnetoresistance at classically large magnetic fields in polycrystalline samples of metals, having open Fermi surfaces, was explained by Lifshits and Peschansky in 1958. The other phenomenon is the quantum linear magnetoresistance appearing in metals, or semimetals, with a small concentration of carriers and a small effective mass, when only the lowest Landau band participates in the conductivity. Manifestations of this unusual phenomenon in different materials are described.

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1. Discovery of the quantum magnetoresistance

In 1928 Kapitza [1], working in the Royal Mond Laboratory of Sir Ernest Rutherford in Cambridge (figure 1), published a paper in the Proceedings of the Royal Society, where he described his device for creating pulsed magnetic fields up to 32 T (320 000 Oe), fantastically high for that time, and measurements of magnetoresistance of bismuth. The latter varied linearly with magnetic field at high fields (figure 2). The next year Kapitza found the linear dependence in a large number of other metals [2]. His discovery became known as 'Kapitza's linear law'.

This phenomenon remained a mystery for many years, since the theory predicted a quadratic dependence at small fields and saturation at higher fields (see, e.g., [3]):

$$\Delta\rho \sim \begin{cases} \rho_0(\Omega\tau)^2 & \Omega\tau \ll 1 \\ \rho_0 & \Omega\tau \gg 1. \end{cases} \tag{1}$$

It existed until the end of the 1950s, when the Ukrainian theorist Ilia Lifshits with his associates published a series of papers on galvanomagnetic phenomena in classically strong magnetic fields, when the Larmor radius of an electron moving through a metal in a magnetic field

Figure 1. Peter Kapitza at the time of his work in Cambridge at the laboratory of Sir Ernest Rutherford.

becomes smaller than its mean free path. In a single crystal the dependence of the resistance on magnetic field is extremely anisotropic. In order to compare these predictions with Kapitza's measurements, performed on polycrystalline samples, an averaging was performed. It showed that in cases when the metal had an energy spectrum of electrons with open Fermi surfaces, the polycrystal would have a magnetoresistance linear in magnetic field [4]. This fitted Kapitza's data well, and seemed to solve the mystery, since most of the metals have an open Fermi surface. People forgot that the first metal, where Kapitza observed a linear magnetoresistance, was bismuth, which has small and closed Fermi surfaces, and hence, this reasoning does not apply.

Much later, in 1969, I constructed an exact quantum theory of galvanomagnetic phenomena in metals, using the field theory technique [5]. Due to the so-called 'Landau quantization' of the electron motion in a magnetic field, all thermodynamic and kinetic coefficients acquire small corrections, varying periodically with magnetic field (figure 3). For conventional metals these corrections are small, due to the fact that the 'Landau bands', essential for physical properties, have a large quantum number. Since, however, my technique was very general, I also considered the 'extreme quantum case' where the distance between the bands, proportional to the magnetic field, was so large that all electrons occupied only the lowest band, leaving the others empty (figure 4). The resistance for this case varied linearly

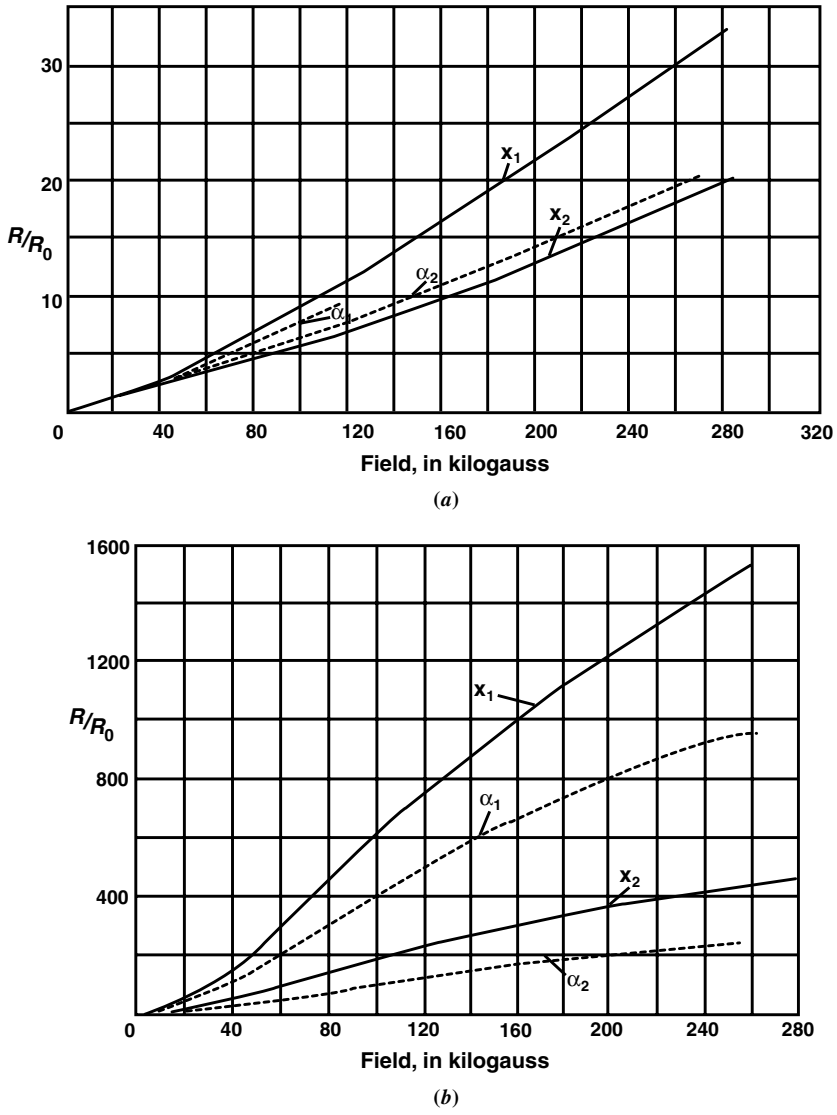

Figure 2. Experimental curves for magnetoresistance perpendicular to the magnetic field of several Bi single crystals, obtained by Kapitza [1]: (*a*) at room temperature, (*b*) at the temperature of liquid nitrogen.

with magnetic field. At that time I believed that this result was 'unphysical', since creation of sufficiently high magnetic fields seemed impossible.

Since this work is the basis of all the subsequent developments, I want to describe some details. I used the isotropic model and the representation of the electron Green function through the eigenfunctions in magnetic field directed along z (units are used with $\hbar = 1$):

$$G^{(0)}_{\alpha\beta}(p_z, p_y, x, x', \omega_m) = \sum_n \frac{\psi_{n\alpha}(x - cp_y/eH)\psi^*_{n\beta}(x' - cp_y/eH)}{\mathrm{i}\omega_m + \mu - \varepsilon_n(p_z)}. \tag{2}$$

The most important feature takes place in strong magnetic fields, such that $(eH/m^*c)\big/\big(p_0^2/2m^*\big) \gg 1$ (p_0 being the Fermi momentum and m^* the effective mass).

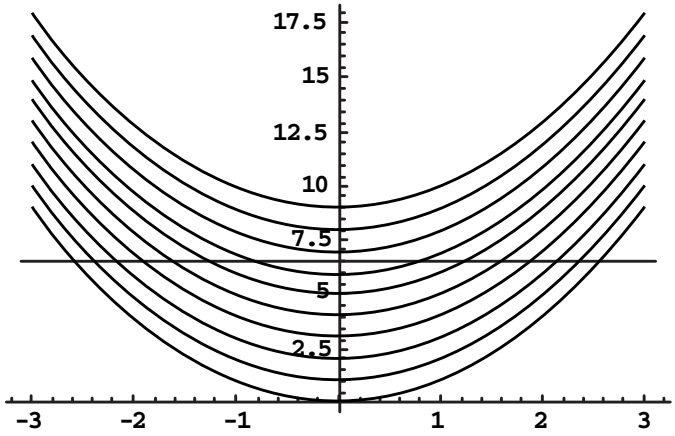

Figure 3. Landau bands in an isotropic model for moderate magnetic fields. The horizontal line corresponds to the chemical potential. Many bands contain electrons.

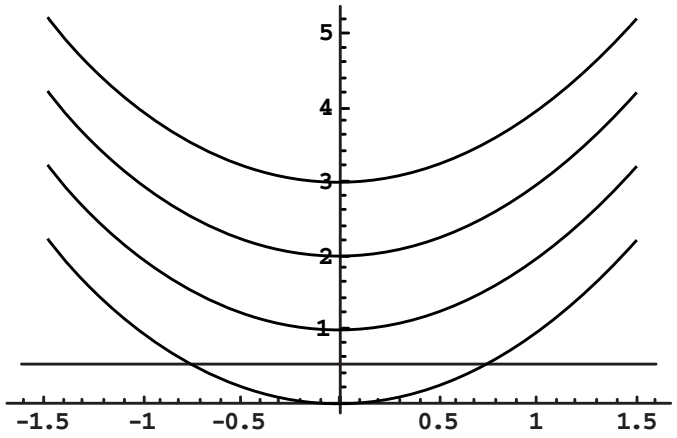

Figure 4. Landau bands in the quantum limit. Only the lowest band contains electrons.

In this case only the lowest Landau band contains electrons. The Born approximation fails in this case, and a summation of diagrams has to be performed, corresponding to figure 5. After that the self-energy due to scattering becomes

$$\sum(\omega_m) = N_i U_0 \left(1 + i\,\mathrm{sgn}(\omega_m) U_0 \frac{eHm^*}{2\pi c p_0}\right)^{-1} \tag{3}$$

(assuming a spin-independent point interaction with impurities: $U(\boldsymbol{r}) = U_0\,\delta(\boldsymbol{r})$; N_i is the impurity concentration). The scattering probability is then $1/\tau = -2\,\mathrm{Im}\Sigma$, and for large fields it becomes

$$\frac{1}{\tau} = \frac{4\pi c p_0 N_i}{eHm^*}. \tag{4}$$

We can call it 'unitary limit' similar to the case without the magnetic field. In this limit the perpendicular to the field components of the resistivity are

$$\rho_{xx} = \rho_{yy} = \frac{N_i H}{\pi n_e^2 e c} \qquad \rho_{xy} = RH = \frac{H}{n_e e c} \tag{5}$$

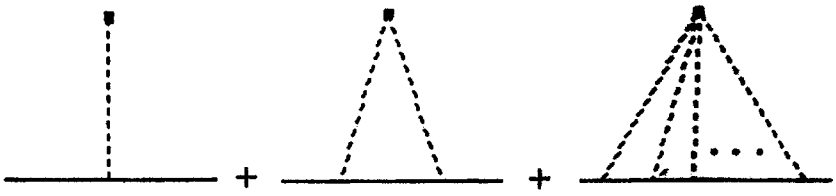

Figure 5. Diagrams to be summed up in the non-Born situation.

where $n_e = eHp_0/(\pi^2 c)$ is the electron density (we assumed the spin splitting to be small compared to the Fermi energy). This means linear magnetoresistance and the Hall constant independent on magnetic field.

The conditions for these results to apply are

$$n_e \ll \left(\frac{eH}{\hbar c}\right)^{3/2} \qquad T \ll \frac{eH\hbar}{m^* c}. \tag{6}$$

The first is for only the lowest Landau band to participate, and the second is for the temperature to be lower than the band splitting. For $H \sim 10$ T the right-hand side of the first inequality is 10^{18} cm^{-3}, whereas the concentration of charge carriers in Bi is of the order of 10^{17} cm^{-3}. Due to the small effective mass in Bi, $m^* \sim 10^{-2}\, m_0$ (m_0 being the free electron mass), the right-hand side of the second inequality is 1000 K. Hence, the conditions for formulae (6) to describe Kapitza's data in Bi were fulfilled even at room temperature. This all meant that in fact Kapitza discovered two new phenomena: the classical linear magnetoresistance in polycrystals of metals with open Fermi surfaces and the *quantum magnetoresistance*, as I called it, in Bi. In 1969 I was not aware of that, and only in 1999, when Chien and his colleagues at Johns Hopkins University published a paper [6], where the authors, being also unaware of Kapitza's discovery, re-established the linear magnetoresistance in Bi, the memory came back to me. I found Kapitza's paper and saw that the criteria (6) are fulfilled.

2. Silver chalcogenides

In 1997 an important discovery was made by experimentalists of the Argonne National Laboratory and the University of Chicago [7]. Studying galvanomagnetic properties of slightly nonstoichiometric silver chalcogenides, $Ag_{2+\delta}Se$ and $Ag_{2+\delta}Te$ with $\delta \sim 0.01$ they found a linear magnetoresistance in fields ranging from 10 Oe to 5.5 T without any sign of saturation (figure 6). This took place at temperatures ranging from 4.5 K to 300 K. The slope of the linear field dependence decreased with increasing temperature in this range approximately 3 times. The Hall constant did not depend on temperature at $T < 100$ K and decreased at higher temperatures.

With ideal stoichiometry, $\delta = 0$, or small nonstoichiometry, at low temperatures these substances are intrinsic semiconductors with a narrow direct gap [8, 9]. The gap is sample dependent and lies in the range of several tens of meV. The effective mass of carriers is of the order of $10^{-2}\, m_0$, where m_0 is the free electron mass [10]. At higher temperatures ($T =$ 133 °C for Ag_2Se) these substances undergo a phase transition into a phase which behaves more like a metal. This also concerns the nonstoichiometric (or doped) compounds, so from the finite value of the Hall constant we can conclude that they are more similar to the high temperature phase of the stoichiometric compounds.

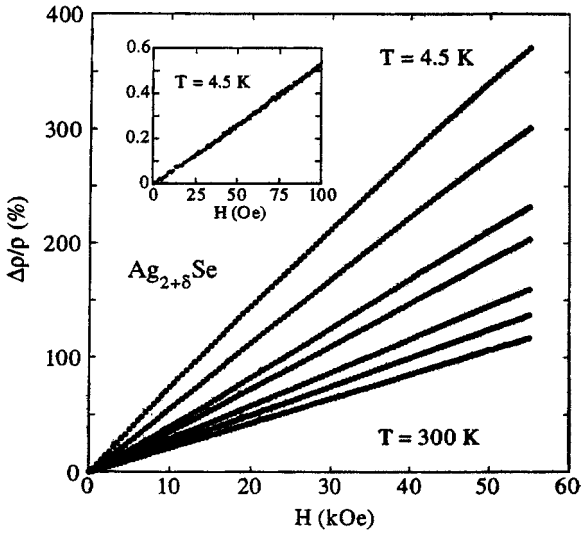

Figure 6. Magnetoresistance curves for $Ag_{2+\delta}Se$ at different temperatures, obtained in [7].

In formula (5) $\rho_{xx} = \rho_{yy}$ is the total resistance, whereas the experimental result [7] can be described as

$$\rho = \rho_0 + aH \tag{7}$$

and this formula is valid not only at magnetic fields of several Tesla but down to 10 Oe. According to what was said before (formula (1)), there is no way to explain such a behaviour assuming that the metal is homogenous. Indeed, if we substitute in the condition (6) the electron concentration obtained from the Hall measurements, $n_e \sim 10^{17}$ cm^{-3}, we get $H > 2$ T.

The only possibility is to assume that the real samples used in the experiments are highly inhomogeneous, so that they contain small regions with a large concentration of excess silver atoms and, correspondingly, higher electron concentration, imbedded into regions with a much smaller electron concentration where the extremal quantum situation takes place. For such a structure one could get the formula (7), however one would have to assume that the concentration in the poorly doped regions has the order of 10^{12} cm^{-3}, i.e. $\sim 10^{-10}$ per atom, or less. This can be obtained from the condition (6) and the experimental result that the dependence of the magnetoresistance deviates from linearity only at $H < 10$ Oe. Since the overall atomic concentration of excess silver atoms is of the order of 1%, it seems that most of these atoms form metallic clusters. For an inhomogeneous system the calculation of the Hall constant requires more precise characterization of the material, and therefore the experimentally obtained 'effective concentrations' of the order of 10^{17} cm^{-3} cannot be calculated unambiguously. However, even these concentrations corresponding to $\sim 10^{-5}$ electrons per atom are much less than the concentration of the excess silver atoms; this shows the trend.

There are, however, other concerns. At small fields the Landau level spacing becomes smaller than T. For a quadratic energy spectrum at 10 Oe it is (in K)

$$\frac{\hbar eH}{m^* c k_B} \sim 10^{-3} \left(\frac{m_0}{m^*}\right) \mathrm{K}$$

where k_B is the Boltzmann constant, and m_0 the mass of the free electron. If one uses the value

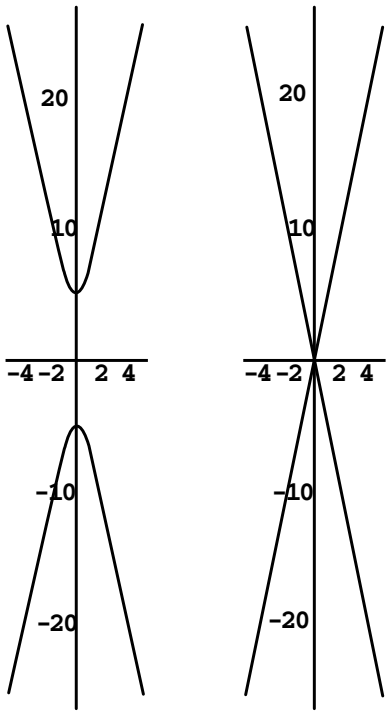

Figure 7. Transition from a narrow-gap semiconductor to a gapless semiconductor.

of $m^*/m_0 \sim 10^{-2}$ given in the literature [10], one gets 10^{-1} K, which is too small, since the linear magnetoresistance at 10 Oe was observed at $T = 4.5$ K.

Since the problem seems completely hopeless for an ordinary approach, some very unusual path has to be taken [11]. In [9] an idea was proposed about the high temperature α-phase of Ag_2Se. Due to the increased mobility of silver ions a substantial disorder appears which creates 'tails' in both, the conduction band and the valence band. Eventually the bands overlap, and the substance becomes metallic. It should be mentioned that according to experimental data the gap is direct. In [12] it was concluded that in Ag_2Te with increasing temperature a phase transition from a narrow-gap semiconductor to a gapless semiconductor takes place (figure 7). One could guess that since the main reason for this change of the spectrum is disorder, the same could happen, as a result of doping, and not of increased mobility (the ions move slowly, and they are always static from the viewpoint of electrons); this conclusion is supported by the finite Hall constant. Under these conditions the nonstoichiometric compound cannot be treated as a semiconductor with carriers in the bands resulting from doping, e.g., Ge and Si, but the start must be made from a different phase, which is closer to a gapless semiconductor [13]. The latter is a substance where at $T = 0$ a completely filled valence band matches an empty conduction band. From the small value of the effective mass [10] it seems more likely that the energy spectrum in both bands is linear. Indeed, since a small effective mass can appear only in some restricted regions of the momentum space, it has to grow with energy (momentum). If the valence band contacts the conduction band, or slightly hybridizes with it, the spectrum becomes even closer to linearity.

The possibility of a linear spectrum was analysed in [15] (type I). It can be a consequence of a cubic symmetry without an inversion centre and can also happen at some random point in

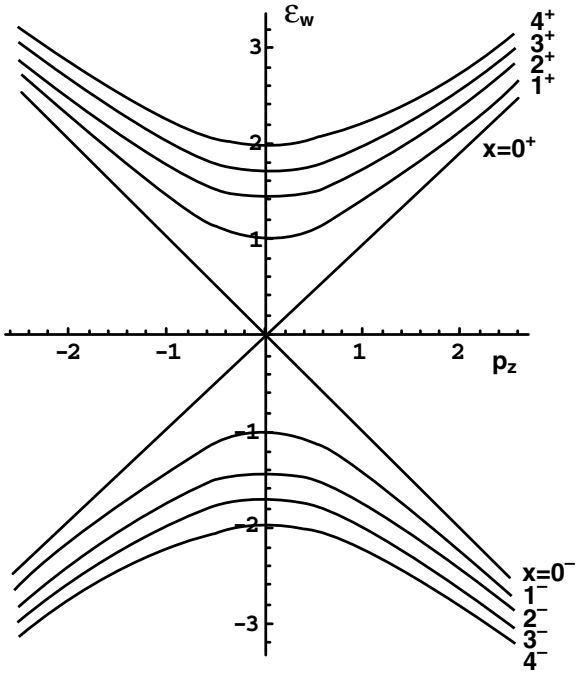

Figure 8. Landau bands for a model of a gapless semiconductor described by the Hamiltonian (8).

the reciprocal space at proper 'tuning', e.g., by pressure or doping. An example of the latter was analysed in [16].

We will consider here the simplest spectrum of this kind, just in order to have an example of what can happen in such a substance. It corresponds to a double representation of a cubic group T or O, and its Hamiltonian can be written as

$$H = \int \psi^{+} v \left(\sigma \left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)\right) \psi \, \mathrm{d}V \tag{8}$$

where σ^i are the Pauli matrices, and $\boldsymbol{p}$ the momentum operators. The velocity v can be assumed of the usual order of magnitude: $v \sim 10^8$ cm s^{-1}. In the absence of the magnetic field we get two branches of the spectrum with energies $\varepsilon = vp$, $-vp$. Suppose that the magnetic field is along z, and we chose the vector potential $A_y = Hx$. The electronic wavefunctions will have two components satisfying the equations

$$\begin{aligned} -\mathrm{i}\frac{\partial}{\partial z}\psi_1 + \left(-\mathrm{i}\frac{\partial}{\partial x} - \frac{\partial}{\partial y} + \mathrm{i}\frac{eH}{c}x\right)\psi_2 &= \frac{\varepsilon}{v}\psi_1 \\ \left(-\mathrm{i}\frac{\partial}{\partial x} + \frac{\partial}{\partial y} - \mathrm{i}\frac{eH}{c}x\right)\psi_1 + \mathrm{i}\frac{\partial}{\partial z}\psi_2 &= \frac{\varepsilon}{v}\psi_2. \end{aligned} \tag{9}$$

Since the equations contain explicitly only x, we will search solutions in the usual form

$$\psi_{1,2} = \psi_{1,2}(x)\, \mathrm{e}^{\mathrm{i}p_y y + \mathrm{i}p_z z}. \tag{10}$$

The eigenvalues of equations (9) are (see figure 8):

$$\varepsilon_n^{(+)} = v\left(p_z^2 + \frac{2eHn}{c}\right)^{1/2} \qquad \varepsilon_n^{(-)} = -v\left(p_z^2 + \frac{2eHn}{c}\right)^{1/2}. \tag{11}$$

Corresponding normalized eigenfunctions have the form:

$$\begin{aligned}
\psi_{n1}^{(+)} &= \frac{1}{\sqrt{2}}\left(1+\frac{p_z}{\left(p_z^2+2neH/c\right)^{1/2}}\right)^{1/2}\psi_n \\
\psi_{n2}^{(+)} &= -\frac{\mathrm{i}}{\sqrt{2}}\left(1-\frac{p_z}{\left(p_z^2+2neH/c\right)^{1/2}}\right)^{1/2}\psi_{n-1} \\
\psi_{n1}^{(-)} &= \frac{1}{\sqrt{2}}\left(1-\frac{p_z}{\left(p_z^2+2neH/c\right)^{1/2}}\right)^{1/2}\psi_n \\
\psi_{n2}^{(-)} &= \frac{\mathrm{i}}{\sqrt{2}}\left(1+\frac{p_z}{\left(p_z^2+2neH/c\right)^{1/2}}\right)^{1/2}\psi_{n-1}
\end{aligned} \tag{12}$$

where ψ_n are the usual normalized eigenfunctions of a free electron in a magnetic field:

$$\psi_n = (2^n n!)^{-1/2}(\beta/\pi)^{1/4}\,\mathrm{e}^{-(\beta/2)[x-(p_y/\beta)]^2}H_n[\sqrt{\beta}(x-p_y/\beta)]. \tag{13}$$

Here $\beta = eH/c$, and H_n are Hermite polynomials.

Formulae (12) describe the eigenfunctions for $n \neq 0$. In the case $n = 0$ the eigenfunctions are

$$\psi_{01}^{(+)} = \theta(p_z)\psi_0 \qquad \psi_{02}^{(+)} = 0 \qquad \psi_{01}^{(-)} = \theta(-p_z)\psi_0 \qquad \psi_{02}^{(-)} = 0. \tag{14}$$

This situation, when for a given spin projection a solution exists only for momenta of a definite sign, happens in different unrelated problems, e.g., in vortex cores.

We assume that in the undoped substance at zero temperature all the negative bands are filled and the positive bands are empty (see [13]). We will assume also that at temperatures and levels of doping under consideration only the bands $\varepsilon_0^{(+)} = v|p_z|$ and $\varepsilon_0^{(-)} = -v|p_z|$ will contain charge carriers. The necessary condition will be established later. Imagine that the doping corresponds to the electron density n_e. This means that the density of electrons in the band $\varepsilon_0^{(+)}$ minus the density of holes in the band $\varepsilon_0^{(-)}$ must be equal to n_e:

$$\frac{eH}{\pi c}\int_0^\infty ([\mathrm{e}^{(vp_z-\mu)/T}+1]^{-1}-[\mathrm{e}^{(vp_z+\mu)/T}+1]^{-1})\frac{\mathrm{d}p_z}{2\pi} = n_e. \tag{15}$$

We took here into account two projections of the electron spin and the fact that the spin splitting $\mu_B H$ can be neglected. It was also important to remember that, according to (11) only the electron states with $p_z > 0$ and the hole states with $p_z < 0$ are available. From condition (12) we obtain

$$\mu = \frac{2\pi^2 n_e c v}{eH}. \tag{16}$$

We see that the chemical potential does not depend on temperature, and this means that at all temperatures the electron system is described by the Fermi distribution.

The condition on temperature replacing (6) is

$$T < v\sqrt{eH\hbar/c}. \tag{17}$$

If we substitute $H \sim 10$ Oe, we get $T < 10$ K, and this corresponds to the measurement conditions of [7]. With larger H the lower temperature boundary also rises, so that at 1 T we get $T < 300$ K, and this is also quite satisfactory. The condition for the electron density remains the same, as given by (6), and it was discussed previously.

We must be aware that for the model under consideration the quantity n_e is only the difference between densities of electrons and holes, entering, as we will see below, the Hall constant. Both types of carriers contribute to the conductivity, and their densities can be considerably larger than n_e. For example, the density of electrons is

$$n_e = \frac{eH}{2\pi^2 c}\int_0^\infty [e^{(vp_z-\mu)/T}+1]^{-1}\,dp_z = \frac{eHT}{2\pi^2 cv}\ln(1+e^{\mu/T}).$$

At low temperatures, when $\mu \gg T$ it is equal to n_e but at higher temperatures and magnetic fields it becomes much larger.

I will not describe all the details of this calculation. The result for the components of the conductivity tensor is

$$\sigma_{xx} = \sigma_{yy} = \frac{1}{2\pi}\left(\frac{e^2}{\varepsilon_\infty v}\right)^2 \ln\varepsilon_\infty \frac{ecN_i}{H} \qquad \sigma_{xy} = \frac{n_e ec}{H}. \tag{18}$$

It was assumed that the scatterers are ions with a screened Coulomb potential. The screening proved to be weak ($\kappa \ll (eH/c)^{1/2}$), provided that ε_∞, the dielectric constant of the ion cores, is large. The corresponding component of the resistivity tensor is

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}. \tag{19}$$

It is most likely that $(e^2/\varepsilon_\infty v)^2 N_i \ll n_e$, and hence

$$\rho_{xx} = \rho_{yy} = \frac{1}{2\pi}\left(\frac{e^2}{\varepsilon_\infty v}\right)^2 \ln\varepsilon_\infty \frac{N_i}{ecn_e^2}H \qquad \rho_{xy} = \frac{H}{n_e ec}. \tag{20}$$

As we argued previously, the resistivity (20) is only a part of the total resistivity, and is due to regions with very small concentration of electrons (term aH in (7)). It indeed depends linearly on H. The interesting feature is that it formally does not depend on temperature. From experimental data [7] we see that it, actually, has to depend on temperature, decreasing approximately three times, as the temperature varies from 4.5 K to 300 K. This rather small decrease of ρ, instead of a marked increase, demonstrates that it is not due to appearance of phonon scattering but is most likely associated with the change of doping (n_e). This is qualitatively confirmed by the fact that the decrease with temperature of the observed resistivity and of the Hall constant starts at the same temperature: 60–70 K in $Ag_{2+\delta}Se$, and 100–130 K in $Ag_{2+\delta}Te$ (figure 9). As we argued already, in an inhomogeneous sample with an unknown internal structure the 'effective' Hall constant cannot be calculated. The electron concentration per atom obtained from it has to be somewhere between $\delta \sim 10^{-2}$ and the limiting value from our estimate (6), i.e. $\sim 10^{-10}$. As we mentioned, the experimental value was $\sim 10^{-5}$.

The fact that both, the resistivity and the Hall constant, decrease with temperature corresponds qualitatively to formulae (20). Since the doping mechanism is rather unusual it is difficult to predict the temperature dependence of n_e. Most likely, it will grow with temperature but much slower than the familiar exponent; this corresponds to the observations. Since the resistance depends not only on n_e but also on N_i, it is hard to compare at this stage samples with different δ.

3. Linear magnetoresistance from small electron groups

Linear magnetoresistance was also found at large fields and low temperatures in rare earth diantimonides with large Fermi surfaces [17]. The latter can be deduced from the Shubnikov–de Haas oscillations which were observed simultaneously with the linear magnetoresistance

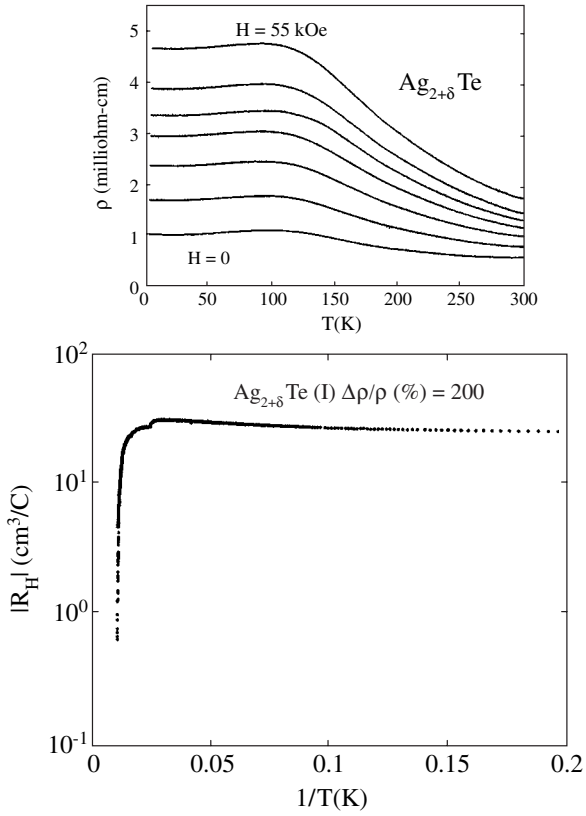

Figure 9. Temperature dependence of the resistance at several magnetic fields and the Hall constant for $Ag_{2+\delta}Te$.

at large fields and lead to an estimate of the electron atomic concentration of the order of 1. Measurements of the Hall constant (S Bud'ko, private communication) lead to the same conclusion. The classical theory at large fields [4, 18] predicts in this case a saturation of resistance, or a quadratic growth with H for specific orientations. In the same paper [17] different possible explanations of the linear dependence were discussed with the conclusion that all of them are unlikely. Since at present no details of the electron energy spectrum of these substances are known, we will propose a model and demonstrate that the linear dependence can be due to quantum magnetoresistance, which can dominate under certain conditions.

Imagine that apart from the large main part of the Fermi surface, it contains a small pocket, for which the conditions (6) are fulfilled. Since this requires a small effective mass, the most natural would be the vicinity of a transition, where a gap between two bands appears at some momentum. At the transition point the bands touch each other; close to the matching point the spectrum is linear, and hence the effective mass is zero. Close to the transition the mass remains small. Such a situation is more common in layered metals, e.g., in graphite (the calculation for a graphite-type spectrum was performed in [19], and it also leads to a linear magnetoresistance). Rare earth diantimonides have also a layered structure.

Electrons on the most of the Fermi surface can be treated classically. In [18] it was found that the *classic* conductivity tensor for an odd metal in a strong magnetic field, such that $\Omega\tau \gg 1$, where τ is the scattering time, and $\Omega = eH/(m^*c)$ the Larmor frequency (m^* is the

'cyclotron mass', usually $m^* \sim m_0$), has the form

$$\sigma_{ik}^c = \begin{pmatrix} \gamma^2 a_{xx}^c & \gamma a_{xy}^c & \gamma a_{xz}^c \\ \gamma a_{yx}^c & \gamma^2 a_{yy}^c & \gamma a_{yz}^c \\ \gamma a_{zx}^c & \gamma a_{zy}^c & a_{zz}^c \end{pmatrix} \tag{21}$$

where $\gamma = (\Omega\tau)^{-1} \ll 1$, and a_{ik}^c are of the order of the conductivity at zero field.

The conductivity in the quantum limit for the isotropic case is

$$\sigma_{ik}^q = \frac{N_i ec}{\pi H} \tag{22}$$

(see [5]). In the general case the quantum conductivity of the small pocket can be presented as $\sigma_{xx}^q = c_i \gamma a_{xx}^q$, $\sigma_{yy}^q = c_i \gamma a_{yy}^q$, where c_i is the atomic concentration of scattering defects, and the field-independent constant a_{ik}^q has the same order of magnitude as the a_{ik}^c in (21). The conductivities from both electron groups add up. In the case, if not only $\Omega\tau \gg 1$ but also $c_i \Omega\tau \gg 1$, σ_{xx}^q dominates over σ_{xx}^c.

Since $\tau \propto c_i^{-1}$, this condition depends only on H, and can be presented, as $Q_{\text{eff}} r_H^2 / a^4 \ll 1$, where Q_{eff} is the electron scattering cross section, $r_H = (c/eH)^{1/2}$ the magnetic radius, and a the interatomic distance. Due to weakness of the pseudopotential for the electron–impurity interaction, this condition can be fulfilled at accessible fields, e.g., if $Q_{\text{eff}} \sim 10^{-18}\,\text{cm}^2$, $H > 10$ T. The resistivity is in this case (according to Onsager's principle, $\sigma_{xy}^c = -\sigma_{yx}^c$)

$$\rho_{xx} = (\sigma_{ik})_{xx}^{-1} = \sigma_{yy}^q \big/ \left(\sigma_{xy}^c\right)^2 \tag{23}$$

and similarly for ρ_{yy}.

We substitute the anisotropic formula corresponding to (22) for σ_{ik}^q and $\sigma_{xy}^c = (n_e - n_h)ec/H$, as obtained in [18], where n_e and n_h are the densities of electrons and holes proportional to the volumes of the momentum space surrounded by parts of the Fermi surface, corresponding, respectively, to smaller and larger energies inside. Then we obtain a linear magnetoresistivity

$$\rho_\alpha(\boldsymbol{n}) = \frac{f_\alpha(\boldsymbol{n}) N_i H}{\pi (n_e - n_h)^2 ec} \tag{24}$$

where $\rho_\alpha(\boldsymbol{n})$ are the main values of the resistivity tensor in the plane normal to $\boldsymbol{n} = \boldsymbol{H}/H$, and $f_\alpha(\boldsymbol{n})$are some functions of the order of unity.

For example, in case of an axially symmetric spectrum

$$\varepsilon = \frac{p_x^2 + p_y^2}{2m_x} + \frac{p_z^2}{2m_z} \tag{25}$$

the Landau bands are

$$\varepsilon_n(p_z) = \frac{e\hbar H\, q^{1/2}(\theta)}{m_x c} \left(n + \frac{1}{2}\right) + \frac{p_z^2}{2m_z q(\theta)} \tag{26}$$

$$q(\theta) = \cos^2\theta + (m_x/m_z)\sin^2\theta \tag{27}$$

and θ is the angle between the magnetic field and the z-axis. The main axes perpendicular to the field are y, normal to the plane defined by z and the field, and the axis x' in that plane, perpendicular to the field. The main values f_α are

$$f_{x'} = q(\theta) \qquad f_y = q^{-1}(\theta). \tag{28}$$

A different situation appears in even metals, where $n_e = n_h$. Then, according to [18],

$$\sigma_{ik}^c = \begin{pmatrix} \gamma^2 a_{xx}^c & \gamma^2 a_{xy}^c & \gamma a_{xz}^c \\ \gamma^2 a_{yx}^c & \gamma^2 a_{yy}^c & \gamma a_{yz}^c \\ \gamma a_{zx}^c & \gamma a_{zy}^c & a_{zz}^c \end{pmatrix}. \tag{29}$$

Under the same condition, as before, namely, $c_1 \Omega\tau \gg 1$, the components of σ_{ik}^q will be larger than all the essential components of σ_{ik}^c, and we obtain

$$\rho_\alpha = \frac{1}{\sigma_\alpha} = \frac{\pi H}{f_\alpha(\boldsymbol{n}) ec N_i}. \tag{30}$$

It is interesting to mention that in this case the resistance decreases with increasing defect concentration. This is due to the fact that in the quantum limit the conductivity across the magnetic field is due to hopping of the electron from a quantized orbit centred along one line parallel to the field to another one, and this can happen only as a result of scattering. In the last case, $n_e = n_h$, the Hall components are of the order of $\gamma^2 a^c$, and the resistivity is just the reciprocal of conductivity. This is also the case for very clean bismuth.

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[13] This reminds the situation in semimetals. The theory can be constructed, if we start not from a simple cubic metal but from a specific type of gapless semiconductor, which looks quite artificial and has definitely a larger energy. However, it becomes energetically favourable at finite deformations (see [14]).
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